

DETERMINATION OF ORTHOMETRIC HEIGHTS FROM GPS AND LEVELLING DATA

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ABSTRACT

One of the major tasks of geodesy is the determination of geoid. This task is getting more crucial due to the development of global positioning systems (GPS). This is due to the fact that GPS provide ellipsoidal heights instead of orthometric heights. To convert ellipsoidal heights into orthometric heights, precise geoid heights are required. Nowadays, the most effective universal technique used for the determination of orthometric heights is the GPS and Levelling technique. This paper focuses on this technique and multiple regression analysis method was used to further determine the geoid undulations. ArcGIS 9.2 software was used for generating the grid map of the area using the corrected orthometric heights obtained by the regression method.

The regression parameters a_0 , x_1 and x_2 were obtained as 1166.721268, -0.00085137265 and -0.00089422771399 respectively. From the analysis, the standard error of estimates S_{x_1} and S_{x_2} associated with x_1 and x_2 were obtained as 0.0001291688765 and 0.0001351270512 respectively. The coefficient of multiple determination R^2 was found to be 0.992049442. the computed F-statistic was 5157.59101, while the value from F-distribution table was 3.97. Hence, the parameter estimators x_1 and x_2 are good estimates of the actual regression parameters x_1 and x_2 .

KEYWORDS: Geoid, Orthometric Heights, Ellipsoidal Heights, Geoid Undulation

INTRODUCTION

Distances observed along plumb lines between equipotential gravitational surfaces and the physical surface are known as orthometric heights. The datum to which orthometric heights are reference to is the geoid which is approximated to the Mean Sea Level (Ghilani and Wolf, 2008). In engineering and survey applications, orthometric heights are required.

The advent of satellite-based positioning technique especially the global positioning system (GPS), which is currently used in a wide range of geodetic and surveying applications, has brought tremendous changes in the processes of precise geodetic control establishment. Data acquisition technique have become more efficient, accuracies greatly improved with new areas of application opened up, orthometric heights can thus be acquired indirectly through ellipsoidal heights from GPS if the geoid over the area is known (Moka and Agajelu, 2006) and (Ghilani and Wolf,2008). Since the ellipsoidal heights from GPS are basically geometric in nature and, therefore, do not reflect the direction of flow under the influence of gravity, heights from GPS are of little or no direct meaning in engineering construction and geodetic applications (Ghilani and Wolf,2008).

To utilize the opportunities provided by this technique, the need for the transformation between ellipsoidal heights and orthometric heights is very important. Using GPS technique, the positions are determined with reference to geocentric World Geodetic System, 1984 (WGS84) reference ellipsoid. Since orthometric heights are determined with reference to the geoid, an accurate geoid model for transforming the ellipsoidal heights from GPS to the highly needed orthometric heights is used. By measuring heights of few GPS stations by spirit levelling and the ellipsoidal heights from GPS, the geoid

undulations can be modelled, which enables the GPS to be used for orthometric heights determination in a much faster and more economical way than terrestrial methods (Rozsa, 1999). Thus a combination of GPS derived ellipsoidal heights and an accurate geoid model provides a new alternative method for orthometric height determination.

Various methods for determination of orthometric heights from GPS and levelling data include among others; Inverse Distance Weighting (IDW) interpolation method, Geometric Interpolation and Multiple Regression Analysis methods. This paper however, emphasized on the application of multiple regression analysis as a tool for determining the orthometric heights in a local area using GPS/ levelling data.

Using the levelling heights and ellipsoidal heights from GPS geoidal undulations (N_{gps}) for all points selected such that they represent the trend of the geoid surface are computed. With the plane coordinates of the points known and using multiple regression analysis, a model is formulated to derive model undulations (N_{model}) from observations. The differences between the GPS undulations and the model undulations (DN) is computed, hence average (DN_{avg}) for several points. Thus orthometric heights are computed by adding the average difference to model undulations and subtracting the result from the ellipsoidal heights.

Problem and Objective of the Study

Heights determined by levelling do not produce true orthometric heights and thus a correction (orthometric correction) must be applied (Soyan, 2005). Since orthometric corrections are a function of gravity data which are insufficient and/or unevenly distributed in Nigeria, orthometric heights cannot be determine directly from spirit levelling. In addition, orthometric heights determine by other techniques such as astrogeodetic techniques are less accurate since assumptions are been made for undulations and the components of the deflection of the vertical for the initial point; in some cases assuming the geoid and the ellipsoid having the same surface normal, disregarding the curvature of the plumb line. Though other accurate techniques exist such as satellite technique, the cost of operation is relatively costly. Therefore, GPS/Levelling technique is hereby advocated as an interim measure to solve the age-long problem of insufficient gravity data and less accurate astrogeodetic approach for orthometric height determination. To convert geodetic heights h (ellipsoidal heights) to elevations H (orthometric heights), the geoid undulations N (geoid heights/geoid separation) must be known (Ghilani and Wolf, 2008).

Unfortunately, the geoid for Nigeria has not been accurately determined. Thus geoid undulations are not readily available. Hence orthometric heights which are necessary for most of the routine geodetic applications are not easily determined. Therefore, there is need to devise a suitable means of solving the problem at hand. This paper is aimed at determining the orthometric heights of points (benchmarks) of an area using GPS and Levelling data to serve practical geodetic applications such as topographical map production, geographic information system (GIS) based studies, engineering applications, etc.

The Concept of Height

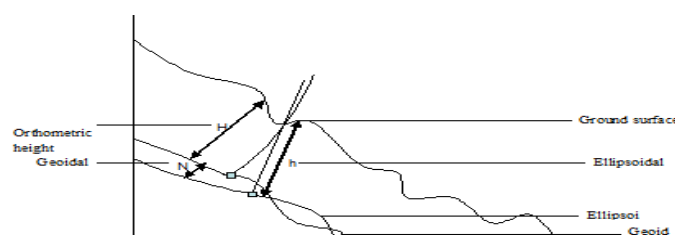


Figure 1: Relationship between the Physical Surface of the Earth, the Geoid and the Ellipsoid

The fundamental relationship between ellipsoidal height (h) obtained from GPS measurements and orthometric heights (H) with respect to a vertical geodetic datum established from spirit levelling data with gravimetric corrections referred to the geoid is given by Heiskanen and Moritz (1969) and Moka and Agajelu (2006) as:

$$N = h - H$$

$$\text{Thus } H = h - N \quad (1)$$

Where N = geoid undulation/ geoid height

h = ellipsoidal height measured along the ellipsoidal normal

H = orthometric height measured along the curved plumb line.

The GPS observed geoidal undulations can be determined as

$$N_{\text{gps}} = h - H \quad (2)$$

Where H = elevations determined by levelling.

The value of N_{gps} obtained in this manner is to be compared with the values of N from a model and the differences computed as (Ghilani and Wolf, 2008):

$$DN = N_{\text{gps}} - N_{\text{model}} \quad (3)$$

To determined the model undulations (N_{model}), we use a base function $f(e,n)$, to functionally represent the geoid undulations (N) as a function of the coordinates of the points observed (Nwilo et al, 2009) as:

$$N = h - H = a_0 + f(e,n) \quad (4)$$

If the geoid is approximated to a flat surface, which is correct over small areas, then we can write an expression for N at any point in terms of some base functions which depend on the coordinates of that point. Hence we have:

$$h_i - H_i = N_i = a_0 + f_i(e,n) \quad (5)$$

The function $f_i(e,n)$ can be expressed in terms of linear combination of some base function as (Opaluwa,2008):

$$f_i(e,n) = e_i x_1 + n_i x_2 \quad (6)$$

Therefore, at any point where ellipsoidal heights from GPS and heights from levelling are known, we can solve for geoid undulation N , using a least square regression model of the form (Featherstone et al, 1998):

$$N_i = h_i - H_i = a_0 + e_i x_1 + n_i x_2 \quad (7)$$

Where a_0 = error term,

x_1, x_2 are tilts of the geoid plane with respect to the WGS84 ellipsoid,

e_i and n_i are the eastings and northings in the same plane coordinate system.

Using multiple regression analysis, the three parameters (a_0, x_1 and x_2) is determined as follows:

$$a_0 = (h_i - H_i)_{\text{mean}} - x_1 \hat{e} - x_2 \hat{n} \quad (8)$$

$$x_1 = \frac{[\sum (e_i - \hat{e})[(h - \hat{H})_i - (h_i - \hat{H}_i)_{\text{mean}}] \sum (n_i - \hat{n})^2] - [\sum (n_i - \hat{n})[(h - \hat{H})_i - (h_i - \hat{H}_i)_{\text{mean}}]]}{[\sum (e_i - \hat{e})^2] \sum (n_i - \hat{n})^2 - [\sum (e_i - \hat{e})(n_i - \hat{n})]^2}$$

$$x_2 = \frac{[\sum (n_i - \hat{n})[(h - \hat{H})_i - (h_i - \hat{H}_i)_{\text{mean}}]]}{[\sum (e_i - \hat{e})^2] \sum (n_i - \hat{n})^2 - [\sum (e_i - \hat{e})(n_i - \hat{n})]^2}$$

$$(\sum(e_i - \hat{e})2\sum(n_i - \tilde{n})2) - [\sum(e_i - \hat{e})(n_i - \tilde{n})]2 \quad (9a)$$

$$X_2 = \frac{[\sum(n_i - \tilde{n})[(h - \hat{H})_i - (h - \hat{H})_{i, \text{mean}}] \sum(e_i - \hat{e})2] - [\sum(e_i - \hat{e})[(h - \hat{H})_i - (h - \hat{H})_{i, \text{mean}}]] [\sum(e_i - \hat{e})(n_i - \tilde{n})]}{(\sum(e_i - \hat{e})2\sum(n_i - \tilde{n})2) - [\sum(e_i - \hat{e})(n_i - \tilde{n})]2} \quad (9b)$$

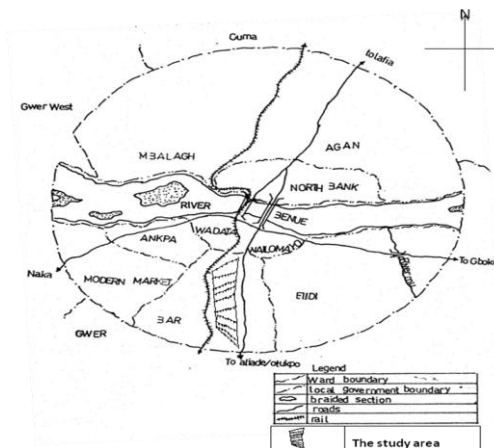
Equation 7 is formed at the points and solved using multiple regression analysis. The solution yields the value of the model parameters; a_0 , x_1 and x_2 , and subsequently the adjusted undulations for the benchmarks. Therefore, the model undulations are substituted in equation 3 to compute the differences in undulations (DN), hence its average for several benchmarks in the area. Using an average DN for the survey area, the orthometric heights are computed as (Ghilani and Wolf, 2008):

$$H = h - (N_{\text{model}} + DN_{\text{avg}}) \quad (10)$$

The Study Area

The area of study is High Level ward in Makurdi local government area, Benue State, north-central Nigeria. The study area has an approximate area of 8.164861742 square kilometres with approximate perimeter of 11.460514 kilometres.

Makurdi, the capital of Benue state is delimited by 16km radius with the centre of the town taken at a control near the post office. It lies between latitudes 7° 28' and 8°00' North, and longitude 8°28' and 8°35' East of Greenwich meridian. It is bounded by Guma local government in the north-east, Tarkaa local government in the east, Gwer local government in the south, Gwer-West local government in the west and Doma local government area of Nassarawa State in the north-west.



Source: Ministry of Lands & Survey Makurdi

Figure 2: Map of Makurdi Local Government Showing the Distribution of Points in the Study Area

Methodology

The methodology adapted in this study involves data acquisition, data processing, as well as numerical investigations.

Data Acquisition

Field surveys were conducted to obtain data used in this study. These include heights obtained from spirit levelling, ellipsoidal heights obtained from GPS as well as positional data using Promark3 GPS receivers. Benchmarks

well-dispersed in the area were observed using both GPS and levelling instrument. The GPS was used in static relative mode for twenty minutes per station with epoch rate of fifteen seconds. Nevertheless, data for existing GPS controls and levelling benchmarks were obtained from the office of the Surveyor General of the Federation, Makurdi zonal office.

Data Processing

Most of the data utilized in this study were digitally processed using computer hardware and software. Least squares adjustment of the levelling network was done using WOLFPACK software while positional data from GPS were done using SURVCARD software.

Equations 2, 3, 7, 8, 9 and 10 were programmed using spreadsheet and solved. Thus, the model parameters (a_0 , x_1 and x_2), GPS observed geoid undulations (N_{gps}), model undulations (N_{model}), difference in undulations ($DN = N_{\text{gps}} - N_{\text{model}}$) and its mean (DN_{avg}), orthometric heights (H) were derived. ArcGIS9.2 software was used for generating the grid map of the area utilizing corrected orthometric heights obtained from equation 10.

Numerical Investigation

The total variation in the dependent variable N_i in equation 2.16 in which N_i is regressed on 'e' and 'n' in a 3- variable model was tested using the coefficient of multiple determination R^2 . This coefficient was calculated using the formula given by Erol and Celik (2003):

$$R^2 = \frac{(x_1 [(h - \hat{H})_i - (h_i - \hat{H}_i)_{\text{mean}}] (e_i - \bar{e}) + x_2 \sum [(h - \hat{H})_i - (h_i - \hat{H}_i)_{\text{mean}}] (n_i - \bar{n}))}{(\sum [(h - \hat{H})_i - (h_i - \hat{H}_i)_{\text{mean}}]^2)^{-1}} \quad (11)$$

The computed coefficient of multiple determinations was 0.992049442. Generally, the higher the value of R^2 the greater the percentage of variation in N_i explained by the regression model which means also that the better the goodness of fit of the regression model to the sample observation. Since this value is greater than zero and less than one ($0 < R^2 < 1$), and also closer to one, it shows that the model is a good fit to the sample observation (Fotopoulos, et al 1999).

Presentation of Results

Since observations were carried out on 85 different stations, the results presented in tabular form became large. Hence, a sample of each table is hereby presented. The results from observations and computations based on equation 2 are shown in the Table 1; while results computed from Equations 8 and 9 are in Tables 2 and 3 respectively.

Table 1: Sample Results From Observations and Equation 2

Station No.	e (m)	n (m)	h (m)	H (m)	$N_{\text{gps}}[h-H] \text{ (m)}$
BSS12	450044.111	855487.444	123.99528	105.612	18.38328
H01	449844.023	854987.401	125.44997	106.433	19.01697
H02	449744.172	854687.125	126.67209	107.170	19.50209
H03	449619.166	854462.244	127.52025	107.603	19.91725
H04	449524.521	854137.587	128.52991	108.116	20.41391
H05	449694.482	853737.591	128.93002	108.316	20.61402
-	-	-	-	-	-
-	-	-	-	-	-
-	-	-	-	-	-
H79	448404.268	852850.098	131.11661	109.877	22.23961
H80	447994.118	852897.453	132.64732	110.046	22.60132
H81	447644.562	852946.198	133.46933	110.569	22.90033
	$\sum = 38121932.03$ $\bar{e} = 448493.318$	$\sum = 72574937.540$ $\bar{n} = 853822.7646$			$\sum = 1813.2296$ $\bar{N} = 21.33211$

Table 2: Sample Data for the Computation of Parameters in Equation 8

Stn/No.	$N_{\text{gps}} - \tilde{N}_{\text{gps}} \text{ (m)}$	$e - \bar{e} \text{ (m)}$	$n - \bar{n} \text{ (m)}$	$(N_{\text{gps}} - \tilde{N}_{\text{gps}})^2 \text{ (m}^2\text{)}$	$(e - \bar{e})^2 \text{ (m}^2\text{)}$	$(n - \bar{n})^2 \text{ (m}^2\text{)}$
BSS12	-2.9488329	1550.790	1664.649	8.69561571	2404958.929	2771057.625
H01	-2.3151429	1350.700	1164.606	5.35988683	1824403.997	1356308.067
H02	-1.8300229	1250.850	864.3304	3.34898396	1564635.729	747067.040
H03	-1.4148629	1125.840	639.4494	2.00183714	1267533.719	408895.535
H04	-0.9182029	1031.200	314.7924	0.84309663	1063379.627	99094.2551
H05	-0.7180929	1201.160	-85.2036	0.51565747	1442794.955	7259.653453
-	-	-	-	-	-	-
-	-	-	-	-	-	-
-	-	-	-	-	-	-
H79	0.90749706	-89.0500	-972.69	0.82355091	7929.902500	946138.6757
H80	1.26920706	-499.200	-925.341	1.61088656	249200.6400	856257.076
H81	1.56821706	-848.756	-876.596	2.45930475	720386.7475	768421.599
				$\Sigma=92.9637785$	$\Sigma=68307748.03$	$\Sigma=62416747.33$

Table 3: Sample Data for the Computation of Parameters in Equation 9

Stn/No.	$(e - \bar{e})(n - \bar{n}) \text{ (m)}$	$(e - \bar{e})(N_{\text{gps}} - \tilde{N}_{\text{gps}}) \text{ (m)}$	$(n - \bar{n})(N_{\text{gps}} - \tilde{N}_{\text{gps}}) \text{ (m)}$
BSS12	2581526.637	-4573.029482	-4908.772984
H01	1573039.688	-3127.075145	-2696.230285
H02	1081151.138	-2289.091515	-1581.744460
H03	719922.8281	-1592.020611	-904.7332581
H04	324614.8673	-946.8536263	-289.0433072
H05	-102343.497	-862.5473882	61.18410362
-	-	-	-
-	-	-	-
-	-	-	-
H79	86618.63223	-80.81261319	-1174.450092
H80	461930.5267	-633.5881644	-1374.693743
H81	744016.6238	-1331.033639	-1574.851242
	$\Sigma=4730893.259$	$\Sigma=53924.61927$	

Table 4: Sample Data for the Computation of Parameters in Equation 10

Stn/No.	$N_{\text{model}} \text{ (m)}$	$DN = N_{\text{gps}} - N_{\text{model}} \text{ (m)}$	$N_{\text{model}} + DN_{\text{avg}} \text{ (m)}$	$H = h - (N_{\text{model}} + DN_{\text{avg}}) \text{ (m)}$
BSS12	18.52315259	-0.13987259	18.52378381	105.4714962
H01	19.14067906	-0.12370906	19.14131028	106.3086597
H02	19.49421944	0.00787056	19.49485066	107.1772393
H03	19.80175206	0.115497937	19.80238328	107.7178667
H04	20.17266356	0.241246438	20.17329478	108.3566152
H05	20.38567069	0.228349308	20.38630191	108.5437181
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-
H79	22.27778832	-0.038178315	22.27841954	109.8381905
H80	22.58463032	0.016689683	22.58526154	110.0620585
H81	22.83864120	0.061688801	22.83927242	110.6300576
		$\Sigma=0.053653453$ $DN=0.0006312170953$		

Thus the values of the Model parameters; a_0 , x_1 , x_2 were solved using multiple regression analysis as shown in Equations 8 and 9. From Tables 2 and 3,

$$\Sigma(e - \bar{e})[(h - \hat{H})_i - (h_i - \hat{H}_i)\text{mean}] = 53924.61927 \quad \Sigma(n - \bar{n})^2 = 62416747.33 \quad \Sigma(n - \bar{n})[(h - \hat{H})_i - (h_i - \hat{H}_i)\text{mean}] = 51790.11711$$

$$\Sigma(e - \bar{e})(n - \bar{n}) = 4730893.259 \quad \Sigma(e - \bar{e})^2 = 68307748.03$$

$$X_1 = -0.0008513726575$$

Similarly, substituting the values in equation 2.18b gives $X_2 = -0.0008942771399$

Substituting X_1 and X_2 in equation 2.17 gives

$$a_0 = 1166.721268.$$

Therefore, with the values of a_0 , x_1 and x_2 , Model 2.16 becomes:

$$N_i = h_i - \hat{H} = 1166.721268 - 0.0008513726575e_i - 0.0008942771399n_i + V_i \quad (12)$$

Where V_i is the error term.

Using Equation 12 the parameters of Equation 3, Model undulations (N_{model}) and Equation 2.19 were computed and presented in Table 4. The last column of table 4 gives the orthometric heights of the points. The gridded contour map and digital elevation model of the elevations obtained from observations (eqn. 2) were presented in figures 3 and 4, while the gridded contour map and digital terrain model of orthometric heights obtained from adjusted undulations (using multiple regression analysis i.e. Model 10) were presented in figure 5 and 6 respectively.

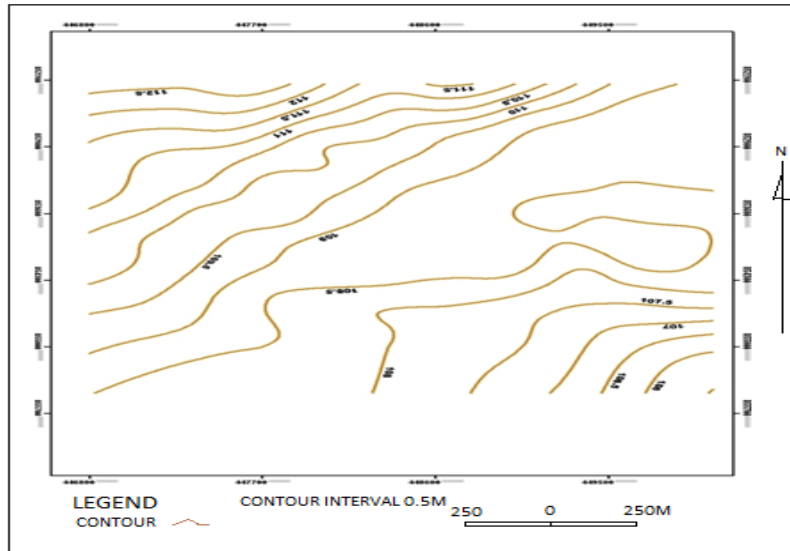


Figure 3: Contour Map of Elevations (\hat{H}) Obtained from Observation

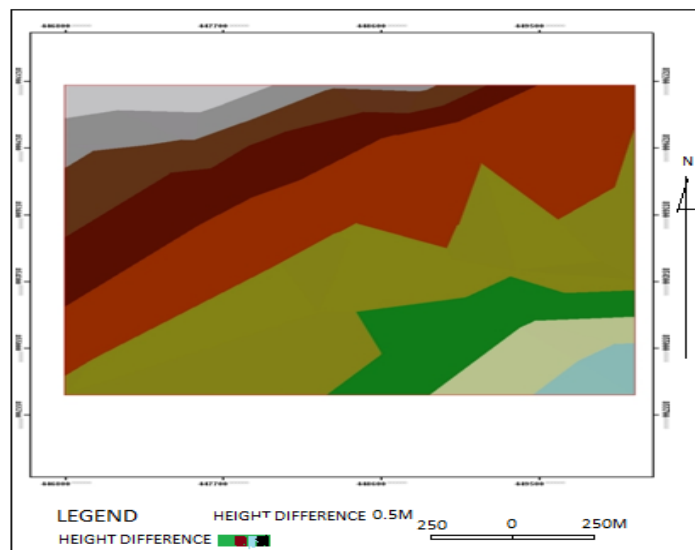


Figure 4: Digital Terrain Model of Elevations (\hat{H}) Obtained from Observation

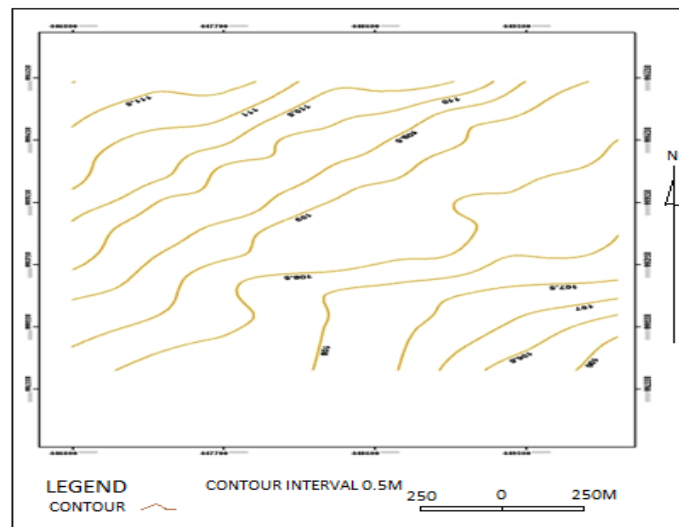


Figure 5: Contour Map Using Orthometric Heights

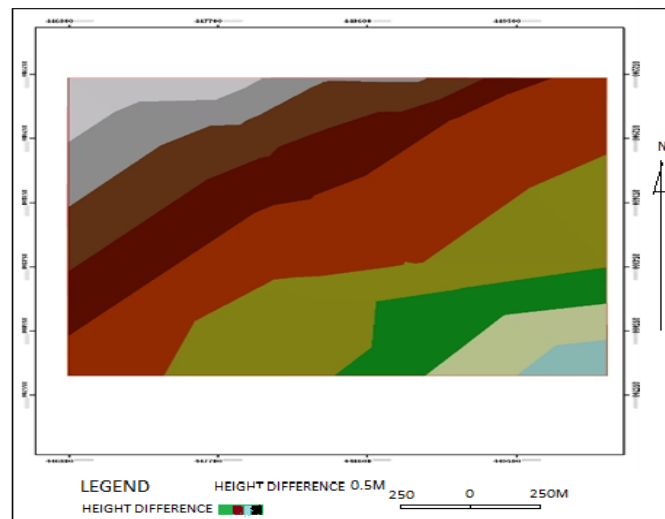


Figure 6: Digital Terrain Model Using Orthometric Heights

Hypothesis Testing

In order to determine the model that will give the best estimate of geoid undulations and subsequently the orthometric height for any arbitrary point in the study area, the following tests were performed;

Test of Significance of Parameter Estimates

The test of significance of parameter estimates is the test of the null hypothesis $H_0: x=\kappa$ and the alternative hypothesis $H_A \neq \kappa$ (where x is the actual regression parameter and κ is the parameter estimator of x).

To test the significance of the difference between the estimated parameter κ and the true or hypothetical regression coefficient x , the ratio of the t-distribution was obtained using the formula:

$$t = (\kappa - x) / S\kappa \quad (13a)$$

$$\text{Therefore, } t_1 = (\kappa_1 - x_1) / S\kappa_1 \text{ and } t_2 = (\kappa_2 - x_2) / S\kappa_2 \quad (13b)$$

Where $S\kappa$ = standard error estimate.

The result of the calculated, t_1 and t_2 are each compared with the critical values of "t" at a given level of

significance (usually 0.05) and degree of freedom N-P. When the value of either t_1 or t_2 exceed the value of critical “t” obtained from table then κ_1 or κ_2 is statistically significant and the null hypothesis is rejected. But if t_1 or t_2 or both are less than or equal to the value of critical “t”, the parameter estimates are not statistically significant.

In this case, the actual regression parameters x_1 and x_2 are unknown, but it is assumed that whatever value of x provided, it is less than κ and the value of $S\kappa$ is relatively small, “t” calculated must be greater than “t” tabulated. Based on this assumption, only the standard error of estimates $S\kappa_1$ and $S\kappa_2$ were computed since x_1 and x_2 are not known.

The standard error of estimate was computed using the formula:

$$S^2\kappa_1 = (\Sigma(N-\tilde{N})^2)(N-P)^{-1} \times (\Sigma(n-\tilde{n})^2)(\Sigma(e-\bar{e})^2\Sigma(n-\tilde{n})^2 - [\Sigma(e-\bar{e})(n-\tilde{n})]^2)^{-1} \quad (13c)$$

$$S^2\kappa_2 = (\Sigma(N-\tilde{N})^2)(N-P)^{-1} \times (\Sigma(e-\bar{e})^2)(\Sigma(e-\bar{e})^2\Sigma(n-\tilde{n})^2 - [\Sigma(e-\bar{e})(n-\tilde{n})]^2)^{-1} \quad (13d)$$

$$\text{Thus, } S\kappa_1 = \sqrt{S^2\kappa_1} \text{ and } S\kappa_2 = \sqrt{S^2\kappa_2}$$

Where N=number of observations, P= number of parameter estimates

$$(N-\tilde{N}) = (h-\hat{H})_i - (h_i-\hat{H}_i)_{\text{mean}}$$

Using the values from Table 2 and 3, $\Sigma(N-\tilde{N})^2 = 92.96377805$,

$$\Sigma(e-\bar{e})^2 = 68307748.03, \Sigma(n-\tilde{n})^2 = 62416747.33,$$

$$\Sigma(e-\bar{e})^2\Sigma(n-\tilde{n})^2 - [\Sigma(e-\bar{e})(n-\tilde{n})]^2 = 4.241166098 \times 10^{15}$$

$$\text{Therefore, } S^2\kappa_1 = (92.96377805/82) \times (62416747.33/4.241166098 \times 10^{15})$$

$$S^2\kappa_1 = 1.668459866 \times 10^{-8}, \text{ hence } S\kappa_1 = (1.668459866 \times 10^{-8})^{1/2} = 0.0001291688765$$

$$\text{Similarly, } S^2\kappa_2 = (92.96377805/82) \times (68307748.03/4.241166098 \times 10^{15})$$

$$S^2\kappa_2 = 1.825931998 \times 10^{-8}, \text{ hence } S\kappa_2 = (1.825931998 \times 10^{-8})^{1/2} = 0.0001351270512$$

Since the standard error is small for both $S\kappa_1$ and $S\kappa_2$, it therefore shows that the parameters κ_1 and κ_2 are good estimators of the actual regression parameters x_1 and x_2 .

Coefficient of Multiple Determinations (R^2)

The total variation in the dependent variable N_i in Equation 7 in which N_i is regressed on “e” and “n” in a 3-variable model was tested using the coefficient of multiple determination (R^2). This coefficient was computed using equation 11. Using values obtained from equations 8, 9 and table 3;

$$R^2 = 0.992049442$$

Generally, R^2 is expected to be greater than zero but less than one ($0 < R^2 < 1$). This implies that about 99% of the variation in the dependent variable N_i can be explained by the regression model with two independent variables x_1 and x_2 . Therefore, the model is a good fit to the sample observation.

Test of Overall Significance of the Regression

The overall significance of the regression was tested using the ratio of the explained variance to the unexplained variance. The F-ratio (statistic) was used to perform the test and its formula for 3-variable regression model is given as:

$$F_{K-1, N-K} = (R^2)^2(k-1)^{-1} + (1-R^2)^2(N-K)^{-1} \quad (14)$$

Where R^2 = coefficient of multiple determination

$k-1$ and $N-k$ are degree of freedoms

K =number of parameter estimated

N =number of observations.

Using equation 14;

$$F_{(2-1),(85-2)} = 5157.59101$$

The tabulated F-ratio, $F = 3.97$ at 0.05 level of significance with degree of freedom **1** and **83**. Since the value of the calculated F-ratio exceeds the value of the tabulated F-ratio, the parameter estimates κ_1 and κ_2 are not both equal to zero, and that the coefficient of multiple determinations is significantly different from zero.

Analysis of Results

Student t-test (determined by standard error of estimates $S\kappa$), R^2 and F-test were performed for significance test of fit for model 12. The standard error of estimates $S\kappa_1$ and $S\kappa_2$ obtained were 0.0001291688765 and 0.0001351270512 respectively. The R^2 obtained was 0.992049442 and the computed F-statistic was 5157.59101 while the value from F-distribution table is 3.97; hence the parameter estimators' κ_1 and κ_2 are good estimators of the actual regression parameters x_1 and x_2 . In addition, 99% of the variations in the dependent variable N_i can be explained by the regression model with two dependent variables x_1 and x_2 . More so, the parameter estimates κ_1 and κ_2 and the coefficient of multiple determination R^2 are statistically significant since F-calculated exceed F-tabulated.

Nevertheless, since the entire test performed shows that the model is a good fit to the sample observations, the orthometric heights determined thereof (from equation 10) are significantly closer to their true values. The gridded contour map and the digital terrain models generated from the exercise (figures 5 and 6) further confirmed these results.

CONCLUSIONS

Orthometric heights determination using GPS and Levelling data (by the method of multiple regression analysis) has been discussed. The possibility of using ellipsoidal heights from GPS and Levelling heights from spirit levelling for computing GPS undulations as postulated by Ghilani and Wolf, (2008) was explained, while the potentials of multiple regression analysis for modelling the undulations was also presented. From the foregoing analysis, a regression model which accounts for about 99% of the variations in the dependent variable is a better model for large scale-engineering applications, topographical mapping, GIS based studies; although it may not be very good enough for precise geodetic applications such as space research, geodynamic applications, etc. The overall result from model 12 however, indicates that the objective of this research, which is to meeting the needs of GIS, engineering and topographical applications, has been achieved. It is therefore concluded that the determination of orthometric heights from GPS and Levelling data (by the method of multiple regression analysis) is a promising alternative to the age-long problem of insufficient gravity data for national orthometric heights determination in Nigeria.

REFERENCES

1. Erol, B. and Celik, R.N.(2003). Investigation on Local Precise Geoid Determination using GPS and Levelling Data. International Symposium of Modern Technologies Education and Professional Practice Globalizing Word. Sofia, Bulgaria.

2. Featherstone, W.E., Denith, M.C. and Kirby, J.F. (1998). Strategies for Accurate Determination of Orthometric Heights from GPS. *Survey Review*. January, PP267-278.
3. Fotopoulos, G., Kotsakis, C. and Sideris, M.G.(1999). Evaluation of Geoid Models and their Use in Combined GPS/Levelling/Geoid Height Network Adjustment. Technical Reports of the Department of Geodesy and Geoinformatics, University Stuttgart.November4.
4. Ghilani, C. D., and Wolf, P.R. (2008). *Elementary Surveying: An Introduction to Geomatics*.12th edition, Pearson Education Ltd, London.
5. Heiskanen, W. A. and Moritz, H. (1969). *Physical Geodesy*. W.H. Freeman, Austria.
6. Moka, E.C. and Agajelu, S.I. (2006). On the Problems of Computing Orthometric Heights from GPS Data. *Proceedings of the first International Workshop on Geodesy and Geodynamics*. November. PP 85-91.
7. Opaluwa, Y.D. (2008). *Determination of Optimum Geometrical Interpolation Technique for Modelling Local Geoid and Evaluation of GPS-Derived Orthometric Height*. Unpublished M.Sc. Dissertation, Department of Surveying and Geoinformatics, University of Lagos, Akoka, Nigeria.
8. Rozsa, S. (1999). Geoid Determination for Engineering Purposes in Hungary. *Proceedings of the International Students' Conference on Environmental Development and Engineering*, pp 125-132, Zakopane.
9. Soyan, M. (2005). *A Cost Effective GPS Leveling Method Versus Conventional Leveling Methods For Typical Surveying Applications*, Yildiz Technical University, Istanbul, Turkey.

